Package ‘gyro’

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Type Package

Title Three-Dimensional Hyperbolic Geometry

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Description Hyperbolic geometry in the hyperboloid model, with emphasis on the 3D case. The methods are based on the gyrovector space theory developed by A. A. Ungar that can be found in the book 'Analytic Hyperbolic Geometry: Mathematical Foundations And Applications' <doi:10.1142/5914>.

License GPL-3

Encoding UTF-8

Imports rgl, Rvcg, Morpho, purrr, cxhull (>= 0.3.0), grDevices, rstudioapi, clipr

Suggests rmarkdown, knitr, trekcolors, uniformly, arrangements

URL https://github.com/stla/gyro

BugReports https://github.com/stla/gyro/issues

RoxygenNote 7.1.2

VignetteBuilder knitr

NeedsCompilation no

Repository CRAN

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R topics documented:

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changesOfSign *Changes of sign*

Description

Sometimes, the coordinates of the vertices of a polyhedron are given with changes of sign (with a symbol **+/-**). This function performs the changes of sign.

Usage

```
changesOfSign(M, changes = "all")
```

Arguments

M	a numeric matrix of coordinates of some points (one point per row)
changes	either the indices of the columns of M where the changes of sign must be done, or "all" to select all the indices

Value

A numeric matrix, M transformed by the changes of sign.

Examples

```
library(gyro)
library(rgl)
## ~~ rhombicosidodecahedron ~~##
phi <- (1 + sqrt(5)) / 2
vs1 <- rbind(
  c(1, 1, phi^3),
  c(phi^2, phi, 2 * phi),
  c(2 + phi, 0, phi^2)
)
vs2 <- rbind(vs1, vs1[, c(2, 3, 1)], vs1[, c(3, 1, 2)]) # even permutations
vs <- changesOfSign(vs2)
open3d(windowRect = c(50, 50, 562, 562), zoom = 0.65)
plotGyrohull3d(vs)
```

gyroABt — *Point on a gyroline*

Description

Point of coordinate `t` on the gyroline passing through two given points `A` and `B`. This is `A` for `t=0` and this is `B` for `t=1`. For `t=1/2` this is the gyromidpoint of the gyrosegment joining `A` and `B`.

Usage

```
gyroABt(A, B, t, s)
```

Arguments

`A, B`	two distinct points
`t`	a number
`s`	positive number, the parameter defining the hyperbolic curvature

Value

A point.

gyrodemos — *Examples of the 'gyro' package*

Description

Some examples of hyperbolic polyhedra realized with the 'gyro' package.

Usage

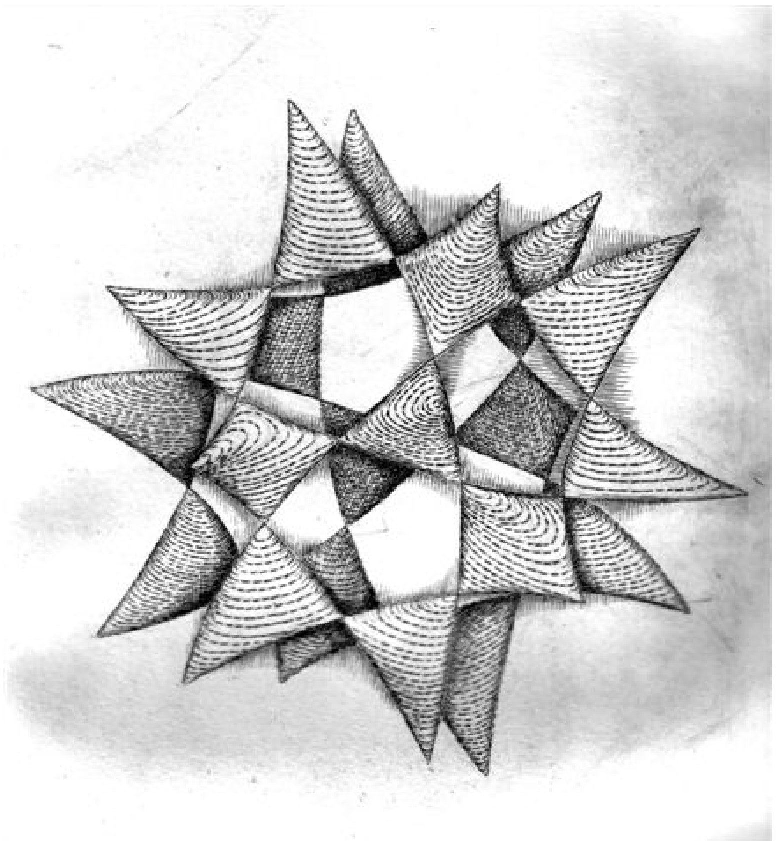
```
gyrodemos()
```

Value

No value. The function firstly copies the demo files in a temporary directory. If you use RStudio, the function opens these files. Otherwise it prints a message giving the instructions to access to these files.

Note

The *BarthLike* file has this name because the figure it generates looks like the Barth sextic (drawing by Patrice Jeener):



`gyrosegment` *Gyrosegment*

Description

Gyrosegment joining two given points.

Usage

```
gyrosegment(A, B, s = 1, n = 100)
```

Arguments

`A, B`	two distinct points (of the same dimension)
`s`	positive number, the curvature
`n`	number of points forming the gyrosegment from `A` to `B`

Value

A numeric matrix with n rows. Each row is a point on the gyrosegment from A (the first row) to B (the last row).

Examples

```
library(gyro)
# a 2D example ####
A <- c(1, 2); B <- c(1, 1)
plot(rbind(A, B), type = "p", pch = 19, xlab = NA, ylab = NA,
     xlim = c(0, 2), ylim = c(0, 2), asp = 1)
AB <- gyrosegment(A, B, s = 0.2)
lines(AB) # this is a piece of an hyperboloid
text(t(A), expression(italic(A)), pos = 1)
text(t(B), expression(italic(B)), pos = 3)

# a 3D hyperbolic triangle
library(rgl)
A <- c(1, 0, 0); B <- c(0, 1, 0); C <- c(0, 0, 1)
s <- 0.3
AB <- gyrosegment(A, B, s)
AC <- gyrosegment(A, C, s)
BC <- gyrosegment(B, C, s)
view3d(30, 30, zoom = 0.75)
lines3d(AB, lwd = 3); lines3d(AC, lwd = 3); lines3d(BC, lwd = 3)
```

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gyrotriangle *Gyrotriangle in 3D space*

Description

3D gyrotriangle as a mesh.

Usage

```
gyrotriangle(
  A,
  B,
  C,
  s = 1,
  iterations = 5,
  palette = NULL,
  bias = 1,
  interpolate = "linear",
  g = identity
)
```

Arguments

A, B, C	three distinct 3D points
s	positive number, the curvature (the smaller, the more curved)
iterations	the gyrotriangle is constructed by iterated subdivisions, this argument is the number of iterations
palette	a vector of colors to decorate the triangle, or NULL if you don't want to use a color palette
bias, interpolate	if palette is not NULL, these arguments are passed to colorRamp
g	a function from [0,1] to [0,1]; if palette is not NULL, this function is applied to the scalars defining the colors (the normalized gyrodistances to the gyrocentroid of the gyrotriangle)

Value

A mesh3d object.

Examples

```
library(gyro)
library(rgl)
A <- c(1, 0, 0); B <- c(0, 1, 0); C <- c(0, 0, 1)
ABC <- gyrotriangle(A, B, C, s = 0.3)
open3d(windowRect = c(50, 50, 562, 562))
view3d(30, 30, zoom = 0.75)
shade3d(ABC, color = "navy", specular = "cyan")

# using a color palette ####
library(trekcolors)
ABC <- gyrotriangle(
  A, B, C, s = 0.5,
  palette = trek_pal("klingon"), bias = 1.5, interpolate = "spline"
)
open3d(windowRect = c(50, 50, 562, 562))
view3d(zoom = 0.75)
shade3d(ABC)

# hyperbolic icosahedron ####
library(rgl)
library(Rvcg) # to get the edges with the `vcgGetEdge` function
icosahedron <- icosahedron3d() # mesh with 12 vertices, 20 triangles
vertices <- t(icosahedron$vb[-4, ])
triangles <- t(icosahedron$it)
edges <- as.matrix(vcgGetEdge(icosahedron)[, c("vert1", "vert2")])
s <- 0.3
open3d(windowRect = c(50, 50, 562, 562))
view3d(zoom = 0.75)
for(i in 1:nrow(triangles)){
  triangle <- triangles[i, ]
  A <- vertices[triangle[1], ]
```

```
  B <- vertices[triangle[2], ]
  C <- vertices[triangle[3], ]
  gtriangle <- gyrotriangle(A, B, C, s)
  shade3d(gtriangle, color = "midnightblue")
}
for(i in 1:nrow(edges)){
  edge <- edges[i, ]
  A <- vertices[edge[1], ]
  B <- vertices[edge[2], ]
  gtube <- gyrotube(A, B, s, radius = 0.03)
  shade3d(gtube, color = "lemonchiffon")
}
spheres3d(vertices, radius = 0.05, color = "lemonchiffon")
```

gyrotube — *Gyrotube (tubular gyrosegment)*

Description

Tubular gyrosegment joining two given 3D points.

Usage

```
gyrotube(A, B, s = 1, n = 100, radius, sides = 90, caps = FALSE)
```

Arguments

`A, B`	distinct 3D points
`s`	positive number, the curvature (higher value, less curved)
`n`	number of points forming the gyrosegment
`radius`	radius of the tube around the gyrosegment
`sides`	number of sides in the polygon cross section
`caps`	Boolean, whether to put caps on the ends of the tube

Value

A `mesh3d` object.

Examples

```
library(gyro)
library(rgl)
A <- c(1, 2, 0); B <- c(1, 1, 0)
tube <- gyrotube(A, B, s = 0.2, radius = 0.02)
shade3d(tube, color = "orangered")

# a 3D hyperbolic triangle ####
library(rgl)
```

```
A <- c(1, 0, 0); B <- c(0, 1, 0); C <- c(0, 0, 1)
s <- 0.3
r <- 0.03
AB <- gyrotube(A, B, s, radius = r)
AC <- gyrotube(A, C, s, radius = r)
BC <- gyrotube(B, C, s, radius = r)
view3d(30, 30, zoom = 0.75)
shade3d(AB, color = "gold")
shade3d(AC, color = "gold")
shade3d(BC, color = "gold")
spheres3d(rbind(A, B, C), radius = 0.04, color = "gold")
```

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plotGyrohull3d *Hyperbolic convex hull*

Description

Plot the hyperbolic convex hull of a set of 3D points.

Usage

```
plotGyrohull3d(
  points,
  s = 1,
  iterations = 5,
  n = 100,
  edgesAsTubes = TRUE,
  verticesAsSpheres = edgesAsTubes,
  edgesColor = "yellow",
  spheresColor = edgesColor,
  tubesRadius = 0.03,
  spheresRadius = 0.05,
  facesColor = "navy",
  bias = 1,
  interpolate = "linear",
  g = identity
)
```

Arguments

`points`	matrix of 3D points, one point per row
`s`	curvature parameter
`iterations`	argument passed to `gyrotriangle`
`n`	argument passed to `gyrotube` or `gyrosegment`, the number of points for each edge
`edgesAsTubes`	Boolean, whether to represent tubular edges

verticesAsSpheres
: Boolean, whether to represent the vertices as spheres

edgesColor
: a color for the edges

spheresColor
: a color for the spheres, if verticesAsSpheres = TRUE

tubesRadius
: radius of the tubes, if edgesAsTubes = TRUE

spheresRadius
: radius of the spheres, if verticesAsSpheres = TRUE

facesColor
: this argument sets the color of the faces; it can be either a single color or a color palette, i.e. a vector of colors; if it is a color palette, it will be passed to the argument palette of gyrotriangle

bias, interpolate, g
: these arguments are passed to gyrotriangle in the case when facesColor is a color palette

Value

No value, called for plotting.

Examples

```
library(gyro)
library(rgl)
# Triangular orthobicopula ####
points <- rbind(
  c(1, -1/sqrt(3), sqrt(8/3)),
  c(1, -1/sqrt(3), -sqrt(8/3)),
  c(-1, -1/sqrt(3), sqrt(8/3)),
  c(-1, -1/sqrt(3), -sqrt(8/3)),
  c(0, 2/sqrt(3), sqrt(8/3)),
  c(0, 2/sqrt(3), -sqrt(8/3)),
  c(1, sqrt(3), 0),
  c(1, -sqrt(3), 0),
  c(-1, sqrt(3), 0),
  c(-1, -sqrt(3), 0),
  c(2, 0, 0),
  c(-2, 0, 0)
)
open3d(windowRect = c(50, 50, 562, 562))
view3d(zoom = 0.7)
plotGyrohull3d(points, s = 0.4)

# a non-convex polyhedron with triangular faces ####
vertices <- rbind(
  c(-2.1806973249, -2.1806973249, -2.1806973249),
  c(-3.5617820682, 0.00000000000, 0.00000000000),
  c(0.00000000000, -3.5617820682, 0.00000000000),
  c(0.00000000000, 0.00000000000, -3.5617820682),
  c(-2.1806973249, -2.1806973249, 2.18069732490),
  c(0.00000000000, 0.00000000000, 3.56178206820),
  c(-2.1806973249, 2.18069732490, -2.1806973249),
  c(0.00000000000, 3.56178206820, 0.00000000000),
```

```
  c(-2.1806973249, 2.18069732490, 2.18069732490),
  c(2.18069732490, -2.1806973249, -2.1806973249),
  c(3.56178206820, 0.00000000000, 0.00000000000),
  c(2.18069732490, -2.1806973249, 2.18069732490),
  c(2.18069732490, 2.18069732490, -2.1806973249),
  c(2.18069732490, 2.18069732490, 2.18069732490))
triangles <- 1 + rbind(
  c(3, 2, 0),
  c(0, 1, 3),
  c(2, 1, 0),
  c(4, 2, 5),
  c(5, 1, 4),
  c(4, 1, 2),
  c(6, 7, 3),
  c(3, 1, 6),
  c(6, 1, 7),
  c(5, 7, 8),
  c(8, 1, 5),
  c(7, 1, 8),
  c(9, 2, 3),
  c(3, 10, 9),
  c(9, 10, 2),
  c(5, 2, 11),
  c(11, 10, 5),
  c(2, 10, 11),
  c(3, 7, 12),
  c(12, 10, 3),
  c(7, 10, 12),
  c(13, 7, 5),
  c(5, 10, 13),
  c(13, 10, 7))
edges0 <- do.call(c, lapply(1:nrow(triangles), function(i){
  face <- triangles[i, ]
  list(
    sort(c(face[1], face[2])),
    sort(c(face[1], face[3])),
    sort(c(face[2], face[3]))
  )
}))
edges <- do.call(rbind, edges0)
edges <- edges[!duplicated(edges), ]
s <- 2
library(rgl)
open3d(windowRect = c(50, 50, 1074, 562))
mfrow3d(1, 2)
view3d(zoom = 0.65)
for(i in 1:nrow(triangles)){
  triangle <- triangles[i, ]
  A <- vertices[triangle[1], ]
  B <- vertices[triangle[2], ]
  C <- vertices[triangle[3], ]
  gtriangle <- gyrotriangle(A, B, C, s)
  shade3d(gtriangle, color = "violetred")
```

```
}
for(i in 1:nrow(edges)){
  edge <- edges[i, ]
  A <- vertices[edge[1], ]
  B <- vertices[edge[2], ]
  gtube <- gyrotube(A, B, s, radius = 0.06)
  shade3d(gtube, color = "darkviolet")
}
spheres3d(vertices, radius = 0.09, color = "deeppink")
# now plot the hyperbolic convex hull
next3d()
view3d(zoom = 0.65)
plotGyrohull3d(vertices, s)

# an example of color palette ####
library(trekcolors)
library(uniformly)
set.seed(666)
points <- runif_on_sphere(50, d = 3)
open3d(windowRect = c(50, 50, 562, 562))
plotGyrohull3d(
  points, edgesColor = "brown",
  facesColor = trek_pal("lcars_series"), g = function(u) 1-u^2
)
```

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