# Weighted Support Vector Machine Formulation

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The original formulation of unweighted SVM with linear kernel is as follows Valdimir and Vapnik (1995):

$$\begin{aligned} \min_{\omega,\xi} \quad & \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{n}(\xi_i + \xi_i^*) \\ \text{s.t.} \quad & y_i - \langle \omega, x_i \rangle - \omega_0 \leq \varepsilon + \xi_i, \\ & \langle \omega, x_i \rangle + \omega_0 - y_i \leq \varepsilon + \xi_i^*, \\ & \xi_i, \xi_i^* \geq 0. \end{aligned}$$

The constant $C > 0$ determines the trade-off between the flatness of $f$ and the amount up to which deviations larger than $\varepsilon$ are tolerated. This corresponds to dealing with a so called $\varepsilon$-insensitive loss function $|\xi|_\varepsilon$ described by

$$|\xi|_\varepsilon = \begin{cases} 0, & \text{if } |\xi| \leq \varepsilon \\ |\xi| - \varepsilon, & o/w. \end{cases}$$

The corresponding weighted SVM with $W_i$ as individual weights:

$$\begin{aligned} \min_{\omega,\xi} \quad & \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{n}W_i(\xi_i + \xi_i^*) \\ \text{s.t.} \quad & y_i - \langle \omega, x_i \rangle - \omega_0 \leq \varepsilon + \xi_i, \\ & \langle \omega, x_i \rangle + \omega_0 - y_i \leq \varepsilon + \xi_i^*, \\ & \xi_i, \xi_i^* \geq 0. \end{aligned}$$

Other kinds of weighted SVMs (with different kernels) have the similar formuation.
Available kernels:

| kernel | formula | parameters |
|---|---|---|
| linear | $\mathbf{u}^\top \mathbf{v}$ | (none) |
| polynomial | $(\gamma \mathbf{u}^\top \mathbf{v} + c_0)^d$ | $\gamma, d, c_0$ |
| radial basis fct. | $\exp\{-\gamma\lvert\mathbf{u} - \mathbf{v}\rvert^2\}$ | $\gamma$ |
| sigmoid | $\tanh\{\gamma \mathbf{u}^\top \mathbf{v} + c_0\}$ | $\gamma, c_0$ |

# References

V Valdimir and N Vapnik. The nature of statistical learning theory. 1995.