

# Package ‘KRIG’

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**Type** Package

**Title** Spatial Statistic with Kriging

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**License** LGPL-3

**Description** Implements different methods for spatial statistics, in particular focused in Kriging based models. We count with different implemented models, simple, ordinary and universal forms of Kriging, co-Kriging and regression Kriging models. Includes, multivariate sensitivity analysis under an approximation designed over reproducing kernel Hilbert spaces and computation of Sobol indexes under this framework.

**URL** <https://github.com/pedroguarderas/KRIG>

**BugReports** <https://github.com/pedroguarderas/KRIG/issues>

**Depends** R (>= 3.3)

**LinkingTo** Rcpp, RcppArmadillo

**RoxygenNote** 6.0.1

**Imports** Rcpp, Rdpack

**Suggests** data.table, colorRamps, plotly, knitr, rmarkdown, testthat

**LazyData** true

**VignetteBuilder** knitr

**RdMacros** Rdpack

**NeedsCompilation** yes

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**Repository** CRAN

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KRIG-package	<i>Spatial Statistics with Kriging</i>
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### Description

Implements different methods for spatial statistics, in particular focused with Kriging based models. We count with different implemented models, simple, ordinary and universal forms of Kriging, co-Kriging and regression Kriging models. Includes, multivariate sensitivity analysis under an approximation designed over reproducing kernel Hilbert spaces and computation of Sobol indexes under this framework.

The linear algebra operations are supported by RcppArmadillo.

### Author(s)

**Maintainer:** Pedro Guarderas <pedro.felipe.guarderas@gmail.com>

Other contributors:

- Daniel Lagos [contributor]
- Andrés Lopez [contributor]

## References

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## See Also

Useful links:

- <https://github.com/pedroguarderas/KRIG>
- Report bugs at <https://github.com/pedroguarderas/KRIG/issues>

## Examples

```
library( KRIG )  
vignette( topic = 'copper_mining_2d', package = 'KRIG' )
```

---

Copper

*Copper mining data*

---

## Description

This is reproduced from the original description for the dataset. A simulation based on a stockpile of mined material in the former Soviet Union. Boreholes have been drilled into the dump. The drill core is cut every 5 meters and assayed for copper and cobalt content in percentage by weight. This is the only three dimensional set of tutorial data. Coordinates are in meters.

## Usage

Copper

**Format**

A data table with 442 and 7 columns.

**a** cod id

**s** sample id

**x1** first coordinate

**x2** second coordinate

**x3** third coordinate

**Z** copper grade

**C** value

**Source**

[www.edumine.com](http://www.edumine.com).

**References**

Dr. Isobel Clark

**Examples**

```
data( 'Copper', packages = 'KRIG' )
```

---

exp\_kernel

*Exponential kernel.*

---

**Description**

Isotropic kernel.

**Usage**

```
exp_kernel(h, sigma = 1, theta = 1)
```

**Arguments**

**h** distance variable.

**sigma** amplitude parameter.

**theta** smoothness parameter.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
h<-0.3
sigma<-1.0
theta<-10
exp_kernel( h, sigma, theta )
```

---

gaussian\_kernel      *Gaussian kernel.*

---

**Description**

Isotropic kernel.

**Usage**

```
gaussian_kernel(h, sigma = 1, theta = 1)
```

**Arguments**

h	distance variable.
sigma	amplitude parameter.
theta	smoothness parameter.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
h<-4
alpha<-2
gaussian_kernel( h, alpha )
```

integrate\_kernel      *Complete kernel integral.*

---

**Description**

This function is part of the routines employed in the sensitivity analysis, calculate the integral in both coordinate  $x$  and  $y$  of the kernel, over the square domain give by the limits  $a$  and  $b$ .

**Usage**

```
integrate_kernel(Kern, a, b, n)
```

**Arguments**

Kern	Kernel function.
a	Inferior limit for the integral in each coordinate.
b	Superior limit for the integral in each coordinate.
n	Number of uniform division to compute the integral.

**Value**

Real value with the integral value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**See Also**

For a complete application you can check the documentation of [Krigvar](#).

---

Kanova      *KANOVA, kernel anova under RKHS approximations.*

---

**Description**

Under an approximation to the sensitivity analysis based in variance computation the different indexes of combinatorial sensitivity values can be computed employing the values of kernel integrals.

**Usage**

```
Kanova(Kernels, Integral, X)
```

**Arguments**

Kernels	data.frame of kernels composed by four columns. <ol style="list-style-type: none"> <li>Kernel name.</li> <li>Inferior limit for integral.</li> <li>Superior limit for integral.</li> <li>Number of steps for discretization of integrals.</li> </ol>
Integral	A list containing the results of kernel integrals of the functions <a href="#">vector_integrate_kernel</a> .
X	matrix containing in each row the coordinate where the one coordinate integrals will be evaluated.

**Value**

List with containing the Gamma 3D array where the different combination variance are stocked and the total matrix variance named Kanova.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**References**

Durrande, N., Ginsbourger, D., Roustant, O. and Carraro, L. (2013). “ANOVA kernels and RKHS of zero mean functions for model-based sensitivity analysis.” *Journal of Multivariate Analysis*, **115**(C), pp. 57-67. Aronszajn, N. (1950). “Theory of reproducing kernels.” *Transactions of the American Mathematical Society*, **68**(3), pp. 337–404. <http://dx.doi.org/10.2307/1990404>.

**See Also**

For a complete application you can check the documentation of [Krigvar](#).

---

Kov *Spatial covariance matrix.*

---

**Description**

To compute a kriging, it is necessary the spatial covariance matrix. The spatial covariance could computed between to sets of points X and Y with different dimension and the result it is not necessarily a square matrix.

**Usage**

Kov(X, Y, Kern, symmetric = FALSE)

**Arguments**

X	First set of spatial points.
Y	Second set of spatial points.
Kern	Kernel function.
symmetric	If result of computation will be a square matrix, the time computation can be improved setting this parameter to TRUE, the default is FALSE.

**Value**

The spatial covariance matrix.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**See Also**

For a complete application you can check the documentation of [Krig](#).

---

Krig

*Kriging computation.*

---

**Description**

Computes the kriging linear estimator for different types of kriging models.

**Usage**

```
Krig(Z, K, k, G, g, type = "ordinary", cinv = "syminv")
```

**Arguments**

Z	Observed values of the spatial process.
K	Covariance matrix computed for the position $X$ where the spatial process $Z$ was observed.
k	Covariance matrix computed for the position $X$ where the spatial process $Z$ was observed and the position $Y$ where the spatial process $Z$ will be predicted.
G	When universal kriging will be computed, this matrix represents the values of the of the functions representing the mean of the process $Z$ , evaluated in the spatial points $X$ where the spatial process was first observed.
g	When universal kriging will be computed, this matrix represents the evaluation of the functions representing the mean over the new position points $Y$ where the spatial process $Z$ will be predicted.
type	Type of kriging model, possible values are: simple, ordinary, universal.



`cinv` Specifies how the inverse of the covariance matrix  $K$  will be computed. Possible values are: `syminv` = symmetric matrix inverse computation, `inv` = usual armadillo inverse computation, `cholinv` = Cholesky based inverse computation, `ginv` = given inverse not necessary to compute inverse at all.

### Value

Depending of the type of analysis the list of results change.

<code>Z</code>	New estimated values for $Z$ .
<code>L</code>	Linear coefficients determined by kriging.
<code>J</code>	Inverse of the covariance matrix.
<code>tau</code>	Factor computed in the ordinary and universal kriging.
<code>alpha</code>	Factor computed in the ordinary kriging.
<code>A</code>	Factor computed in the universal kriging.

### Author(s)

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

### Examples

```
library( KRIG )
vignette( topic = 'simple_kriging', package = 'KRIG' )
vignette( topic = 'ordinary_kriging', package = 'KRIG' )
vignette( topic = 'universal_kriging', package = 'KRIG' )
vignette( topic = 'copper_mining_2d', package = 'KRIG' )
```

---

Krigidx

*Combinatorial variance computation.*

---

### Description

For a given combination this function computes the associated variance for the variable enumerated by the combination values.

### Usage

```
Krigidx(KF, comb, X, Gamma)
```

### Arguments

<code>KF</code>	values of the kernel integral evaluations.
<code>comb</code>	Combination.
<code>X</code>	Points in the grid.
<code>Gamma</code>	Cube with integral results.

**Value**

Real value of sensitivity.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**References**

Durrande, N., Ginsbourger, D., Roustant, O. and Carraro, L. (2013). “ANOVA kernels and RKHS of zero mean functions for model-based sensitivity analysis.” *Journal of Multivariate Analysis*, **115**(C), pp. 57-67. Aronszajn, N. (1950). “Theory of reproducing kernels.” *Transactions of the American Mathematical Society*, **68**(3), pp. 337–404. <http://dx.doi.org/10.2307/1990404>.

**See Also**

For a complete application you can check the documentation of [Krigvar](#).

---

Krigvar

*Combinatorial variance computation.*

---

**Description**

Computation of variance

**Usage**

Krigvar(KF, Gamma)

**Arguments**

KF                    values of the kernel integral evaluations.  
Gamma                Cube with integral results.

**Value**

Real value of sensitivity.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**References**

Durrande, N., Ginsbourger, D., Roustant, O. and Carraro, L. (2013). “ANOVA kernels and RKHS of zero mean functions for model-based sensitivity analysis.” *Journal of Multivariate Analysis*, **115**(C), pp. 57-67. Aronszajn, N. (1950). “Theory of reproducing kernels.” *Transactions of the American Mathematical Society*, **68**(3), pp. 337–404. <http://dx.doi.org/10.2307/1990404>.

**Examples**

```

library( KRIG )
options( stringsAsFactors = FALSE )

kernel_1<-function( x, y ) exp( -0.5*(x-y)^2)
kernel_2<-function( x, y ) exp( -0.7*(x-y)^2)
kernel_3<-function( x, y ) exp( -0.1*(x-y)^2)

Kernels<-data.frame( kernel = c( 'kernel_1', 'kernel_2', 'kernel_3' ),
                    min = c( -1, -1, -2 ),
                    max = c( 1, 1, 2 ),
                    n = c( 100, 100, 100 ) )

n<-20
X<-matrix( c( seq( -1, 1, length.out = n ),
             seq( -1, 1, length.out = n ),
             seq( -2, 2, length.out = n ) ), n, 3 )

KI<-list_integrate_kernel( Kernels, X )
GK<-Kanova( Kernels, KI, X )

f<-function( x ) x[1] + 30 * x[2] + 60 * x[3]
Func<-apply( X, 1, FUN = f )

KF<-solve( GK$Kanova + diag( 1e-8, n, n ), Func )

SbI<-NULL
for ( j in 1:3 ) {
  CB<-combn( 1:3, j )
  for ( l in 1:ncol( CB ) ) {
    SbI<-c( SbI, Krigidx( KF, CB[,l], X, GK$Gamma ) )
    names(SbI)[length(SbI)]<-paste( 'C.', paste( CB[,l], collapse='.' ), sep = '' )
  }
}

Var<-Krigvar( KF, GK$Gamma )

SVar<-sum( SbI / Var )

```

---

linear\_kernel

*Linear kernel*


---

**Description**

Anisotropic kernel defined by the scalar product.

**Usage**

```
linear_kernel(x, y, alpha)
```

**Arguments**

x	first column vector.
y	second column vector.
alpha	amplitude parameter.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
n<-10
x<-matrix( runif( n ), n, 1 )
y<-matrix( runif( n ), n, 1 )
alpha<-0.5
linear_kernel( x, y, alpha )
```

---

list\_integrate\_kernel *Integrals of a list of kernels.*

---

**Description**

This function is part of the routines employed in the sensitivity analysis, computes vector of integrals and complete integrals of kernels specified in the data frame `Kernels`.

**Usage**

```
list_integrate_kernel(Kernels, X)
```

**Arguments**

Kernels	data.frame of kernels composed by four columns. <ol style="list-style-type: none"><li>1. Kernel name.</li><li>2. Inferior limit for integral.</li><li>3. Superior limit for integral.</li><li>4. Number of steps for discretization of integrals.</li></ol>
X	matrix containing in each row the coordinate where the one coordinate integrals will be evaluated.

**Value**

List with one coordinate integrals and complete kernel integrals.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**See Also**

For a complete application you can check the documentation of [Krigvar](#).

---

matern_kernel	<i>Matérn kernel.</i>
---------------	-----------------------

---

**Description**

Isotropic kernel.

**Usage**

```
matern_kernel(h, v = 2, sigma = 1, theta = 1)
```

**Arguments**

h	distance variable.
v	power parameter.
sigma	amplitude parameter.
theta	smoothness parameter.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
h<-4.0
v<-2.0
sigma<-2.0
theta<-100.0
matern_kernel( h, v, sigma, theta )
```

multilog\_kernel      *Multilog kernel.*

---

**Description**

Isotropic kernel.

**Usage**

```
multilog_kernel(h, R = 1)
```

**Arguments**

h                    distance parameter.  
R                    displacement parameter.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
h<-4  
R<-2.3  
multilog_kernel( h, R )
```

---

nat\_cubic\_spline\_kernel  
                          *Natural cubic spline kernel.*

---

**Description**

Isotropic kernel.

**Usage**

```
nat_cubic_spline_kernel(h, R = 1)
```

**Arguments**

h                    distance variable.  
R                    displacement parameter.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
h<-4.0
R<-2.3
nat_cubic_spline_kernel( h, R )
```

---

polynomial\_kernel      *Polynomial kernel*

---

**Description**

Anisotropic kernel defined like a polynomial in the scalar product.

**Usage**

```
polynomial_kernel(x, y, alpha, beta, n)
```

**Arguments**

x	first column vector.
y	second column vector.
alpha	amplitude parameter.
beta	displacement parameter.
n	power parameter.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>. N<-10 x<-matrix( runif( N ), N, 1 )  
y<-matrix( runif( N ), N, 1 ) alpha<-0.5 beta<-2 n<-3 polynomial\_kernel( x, y, alpha, beta, n )

---

spherical\_kernel      *Spherical kernel.*

---

**Description**

Isotropic kernel commonly employed in geostatistic.

**Usage**

```
spherical_kernel(h, phi, theta)
```

**Arguments**

h	distance variable.
phi	amplitude parameter.
theta	smoothness parameter.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
h<-4.0  
phi<-1.0  
theta<-100.0  
spherical_kernel( h, phi, theta )
```

---

square\_kernel      *Square kernel.*

---

**Description**

Isotropic kernel given by the square distance.

**Usage**

```
square_kernel(h, alpha = 1)
```



**Arguments**

h                    distance variable.  
alpha                amplitude parameter.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
h<-4  
alpha<-2  
square_kernel( h, alpha )
```

---

*thin\_plate\_kernel*        *Thin plate kernel.*

---

**Description**

Isotropic kernel.

**Usage**

```
thin_plate_kernel(h, R = 1)
```

**Arguments**

h                    distance variable.  
R                    displacement parameter.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
h<-4.0  
R<-0.5  
thin_plate_kernel( h, R )
```

---

triangular\_kernel      *Triangular kernel.*

---

**Description**

Isotropic kernel defined with the max function.

**Usage**

```
triangular_kernel(h, c = 1, alpha = 1)
```

**Arguments**

h	distance variable.
c	amplitude parameter.
alpha	maximum distance value.

**Value**

Real value.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
h<-0.2
c<-5
alpha<-2
triangular_kernel( h, c, alpha )
```

---

variogram      *Computes the variogram.*

---

**Description**

This useful function is commonly employed in the study of isotropic stationary spatial processes.

**Usage**

```
variogram(Z, X, d)
```

**Arguments**

Z	Vector of observations.
X	Points matrix.
d	Distance function.

**Value**

Variogram vector.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
library( KRIG )
vignette( topic = 'copper_mining_2d', package = 'KRIG' )
```

---

vector\_integrate\_kernel

*One coordinate kernel integral.*

---

**Description**

This function is part of the routines employed in the sensitivity analysis, takes the kernel  $k$  and for each fixed coordinate in  $x$ , the integral in the second variable  $y$ , is computed in the interval  $a$  to  $b$  by taking  $n$  uniform steps.

**Usage**

```
vector_integrate_kernel(Kern, x, a, b, n)
```

**Arguments**

Kern	Kernel function.
x	Column vector with values for the first coordinate of the kernel.
a	Inferior limit for the integral in $y$ .
b	Superior limit for the integral in $y$ .
n	Number of uniform division to compute the integral.

**Value**

Vector with integrals while the  $x$  coordinate is fixed.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**See Also**

For a complete application you can check the documentation of [Krigvar](#).

---

weight\_pow\_dist      *Generic weighted p-distance*

---

**Description**

Many isotropic models can be defined employing a distance. In particular this function implements a distance employing weights and different powers.

**Usage**

```
weight_pow_dist(x, y, w, p)
```

**Arguments**

x	First vector.
y	Second vector.
w	Weights for every coordinate in the vectors.
p	Powers for every coordinate in the vectors. To define a true norm every coordinate has to be greater than 1.

**Value**

Real value of the weighted p-distance.

**Author(s)**

Pedro Guarderas <pedro.felipe.guarderas@gmail.com>.

**Examples**

```
library(KRIG)

n<-1e5
x<-runif( n )
y<-runif( n )
p<-rep(2.5,n)
w<-runif(n)

kw<-weight_pow_dist( x, y, w, p )
nw<-sum( w * abs( x - y )^p )

nw == kw
```

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